

Are Time-Domain Self-Force Calculations Contaminated by Jost Solutions?

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The calculation of the self force in the modeling of the gravitational-wave emission from extreme-mass-ratio binaries is a challenging task. Here we address the question of the possible emergence of a persistent spurious solution in time-domain schemes, referred to as a *Jost junk solution* in the literature, that may contaminate self force calculations. Previous studies suggested that Jost solutions are due to the use of zero initial data, which is inconsistent with the singular sources associated with the small object, described as a point mass. However, in this work we show that the specific origin is an inconsistency in the translation of the singular sources into jump conditions. More importantly, we identify the correct implementation of the sources at late times as the sufficient condition guaranteeing the absence of Jost junk solutions.

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Extreme-mass-ratio inspirals are one of the most important sources of gravitational radiation for the future space-based observatory Laser Interferometer Space Antenna (LISA). They consist of a stellar compact object inspiralling into a massive black hole (MBH; with mass in the range $M = 10^4 - 10^7 M_\odot$). They are long lasting sources that in the last year before plunge can spend of the order of 10^5 cycles inside the LISA frequency band [1]. To extract these signals from the future LISA data stream we require very precise theoretical waveform templates, as the signal-to-noise ratio accumulates slowly with time [2]. To achieve such precision we need a very accurate description of the slow inspiral, which can be seen as due to the action of a local force, the *self force*. The computation of the self force is a challenge since we need to regularize the gravitational perturbations created by the stellar compact object, which is modeled as a point mass. Nevertheless, during the last decade there has been significant progress in the self-force program (see [3, 4] for reviews) and different methods to compute it have been developed. Of particular relevance is the computation of the gravitational self force in the case of a nonrotating MBH, both for circular and eccentric orbits [5, 6], using time-domain methods.

The aim of this letter is to clarify an issue related to time-domain self-force calculations: The appearance of the so-called *Jost junk* solutions discussed recently in [7], which can contaminate time-domain calculations of the self force, limiting their accuracy. Here, following [7], we show that such solutions are not a generic feature of time-domain schemes, but rather a result of the use of trivial zero initial data combined with a particular implementation of the singular sources associated with the point

mass. We then identify the specific ingredients responsible for the eventual appearance of the spurious solution and point out a straightforward solution, eliminating in this way concerns about the generation of this kind of persistent junk contamination when calculating the self force.

The self force is determined by the gravitational perturbations created by the point mass as it orbits the MBH, what we call the retarded field, whose computation is the main numerical task in time-domain self-force schemes. The retarded field is singular at the location of the point mass and needs to be regularized by using analytic expressions for the singular field (see, e.g. [8]). The retarded field is described by a system of coupled wave-type equations that derive from the perturbed Einstein’s equations, which means that their structure depends on the gauge adopted. Not all the gauges are suitable for self-force computations.

The principal part of the wave-type equations that govern the retarded field is common to most gauges. For nonrotating MBHs, each spherical harmonic component obeys decoupled $1+1$ wave equations whose structure is captured by the following model equation:

$$\begin{aligned} &[-\partial_t^2 + \partial_{r^*}^2 - V(r)] \Psi(t, r) = S(t, r) \\ &= f(r) [G(t, r) \delta(r - r_p(t)) + F(t, r) \delta'(r - r_p(t))] \end{aligned} \quad (1)$$

where $f(r) = 1 - 2M/r$, M is the MBH mass, r^* is the *tortoise* coordinate $r^* = r + 2M\ln[r/(2M) - 1]$, and $r_p(t)$ is the particle’s radial motion. This model equation consists of a $1+1$ wave operator $-\partial_t^2 + \partial_{r^*}^2$, a potential term, $V(r)\Psi$, and a singular source term $S(t, r)$. Equation (1) has the form of the master equations that rule perturbations of different fields in Schwarzschild spacetime. They are tied to a particular set of gauges, but in other gauges the principal structure of the equations that govern the dynamics of the perturbations is similar. Therefore, we will use the model Eq. (1) in order to analyze the appearance of the Jost junk solution, as done in [7]. Without

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loss of generality, we deal with the case of circular orbits ($r_p = \text{const}$).

Time-domain schemes address the resolution of Eq. (1) as an initial-boundary value problem, by prescribing initial conditions on a time slice together with appropriate boundary conditions along the evolution. Appropriate initial data (ID) at a finite time are not known for Eq. (1). A common practice consists of setting *trivial* ID, namely

$$\Psi|_{t=t_o} = (\partial_t \Psi)|_{t=t_o} = 0. \quad (2)$$

This ID is inconsistent with the singular structure of the source and corresponds to a solution that is not continuous in time at $t = t_o$. As a consequence, when solving (numerically) Eq. (1) an initial burst of *junk radiation* is produced and one must wait until it has been radiated away in order to compute the self force. This strategy relies on the assumption that *junk radiation* is actually radiated away. In this sense, Ref. [7] addresses the question of whether the use of trivial ID can give rise to spurious solutions persistent in time. To answer this question a double complementary numerical and analytical approach is adopted in [7] that we reanalyze below.

(a) *Numerical approach.* The impact of inconsistent ID is assessed by constructing, first, a solution to Eq. (1) with trivial ID, referred to as $\Psi_{\text{Impulsive}}$. Second, the sources in Eq. (1) are modified to make them compatible with trivial ID. This is achieved by a smooth switch on in time of the sources

$$F_\tau^S(t, r) \equiv \alpha(t, \tau) F(t, r), \quad G_\tau^S(t, r) \equiv \alpha(t, \tau) G(t, r), \quad (3)$$

where $\alpha(t, \tau)$ smoothly interpolates between 0 and 1, at initial and late times, i.e. $\alpha(t_o, \tau) = 0$ and $\alpha(t, \tau) = 1$ (for $t \gg \tau$). The solution obtained with trivial ID and smooth sources (3) is referred to as Ψ_{Smooth} . At late times, $t \gg \tau$, a numerical function Ψ_{Jost}^N is defined as

$$\Psi_{\text{Jost}}^N \equiv \Psi_{\text{Impulsive}} - \Psi_{\text{Smooth}}. \quad (4)$$

The function Ψ_{Jost}^N has the following properties [14]: (i) It is time independent: $\partial_t \Psi_{\text{Jost}}^N = 0, \forall t$. (ii) It has a jump at the particle: $[\Psi_{\text{Jost}}^N]_p = -f_p^{-1} F(t_o, r_p)$, where $f_p \equiv f(r_p)$. (iii) The spatial derivative, $\partial_{r^*} \Psi_{\text{Jost}}^N$, is continuous at $r = r_p$.

(b) *Analytical approach.* Motivated by the numerical approach, one constructs the analytical function Ψ_{Jost}^A as

$$\Psi_{\text{Jost}}^A \equiv \Psi_{\text{Jost}}^{A,-} \Theta_- + \Psi_{\text{Jost}}^{A,+} \Theta_+, \quad (5)$$

where $\Theta_+ \equiv \Theta(r^* - r_p^*)$, $\Theta_- \equiv \Theta(r_p^* - r^*)$, being Θ the Heaviside step function, and $\Psi_{\text{Jost}}^{A,-}$ and $\Psi_{\text{Jost}}^{A,+}$ solve the homogeneous, stationary version of Eq. (1)

$$[\partial_{r^*}^2 - V(r)] \Psi_{\text{Jost}}^{A,\pm} = 0, \quad (6)$$

such that $[\Psi_{\text{Jost}}^A]_p = -f_p^{-1} F(t_o, r_p)$. This means (see below) that Ψ_{Jost}^A satisfies an inhomogeneous version of Eq. (6) with a stationary singular source given by

$-f_p F(t_o, r_p) \delta'(r - r_p)$. Differences between Ψ_{Jost}^N and Ψ_{Jost}^A are shown to vanish within numerical precision in [7]. Thus, we refer to a single *Jost* function, Ψ_{Jost} .

From this analysis we conclude that at late times, when the time switch-on function $\alpha(t, \tau)$ equals 1 and the sources for $\Psi_{\text{Impulsive}}$ and Ψ_{Smooth} coincide, their difference should be a solution to the homogeneous version of Eq. (1). However, this is in conflict with the fact that, as discussed above, Ψ_{Jost} solves a (stationary) version of Eq. (1) with a singular distributional source. Certainly, this contradiction arises from the use of inconsistent sources and ID. But more importantly, it also suggests strongly that, at late times, one is actually solving two different systems for $\Psi_{\text{Impulsive}}$ and Ψ_{Smooth} , namely with different sources. To assess this point we now discuss the implementation of Eq. (1) in [7].

In the *particle-without-particle* (PwP) approach to Eq. (1), introduced in [9, 10] and further discussed in [7, 11, 12], the point mass is placed at the boundary between two integration domains. Then, one solves a homogeneous problem in each domain and the singular sources in Eq. (1) are translated into *jump* conditions at the particle. To illustrate this it is convenient to rewrite Eq. (1) as a first-order hyperbolic system (cf. Refs. [12, 13] for a second-order formulation). To that end, we introduce the fields [15]

$$\phi \equiv \partial_t \Psi, \quad \varphi \equiv \partial_{r^*} \Psi, \quad (7)$$

and Eq. (1) becomes a system for the vector (Ψ, ϕ, φ)

$$\partial_t \Psi = \phi, \quad \partial_t \phi = \partial_{r^*} \varphi - V(r) \Psi - S(t, r), \quad \partial_t \varphi = \partial_{r^*} \phi. \quad (8)$$

In the PwP approach we perform the splitting

$$\Psi = \Psi^- \Theta_- + \Psi^+ \Theta_+, \quad (9)$$

$$\phi = \phi^- \Theta_- + \phi^+ \Theta_+, \quad (10)$$

$$\varphi = \varphi^- \Theta_- + \varphi^+ \Theta_+ + [\Psi]_p \delta(r^* - r_p^*), \quad (11)$$

where the Dirac term of φ follows from consistency between the definition of φ in (7) and the expression for Ψ in (9). Inserting expressions (9)-(11) in system (8) we obtain homogeneous systems of equations for the fields

$$\partial_t \Psi^\pm = \phi^\pm, \quad \partial_t \phi^\pm = \partial_{r^*} \varphi^\pm - V(r) \Psi^\pm, \quad \partial_t \varphi^\pm = \partial_{r^*} \phi^\pm \quad (12)$$

and jump conditions to communicate them across the particle

$$[\Psi]_p(t) = f_p^{-1} F(t, r_p), \quad (13)$$

$$[\phi]_p(t) = f_p^{-1} (\partial_t F)(t, r_p), \quad (14)$$

$$[\varphi]_p(t) = G(t, r_p) - f_p^{-1} (\partial_{r^*} F)(t, r_p). \quad (15)$$

This whole system, i.e. homogeneous Eqs. (12) and jumps (13)-(15), is equivalent to the original Eq. (1). Two remarks are in order, regarding the system (12)-(15). First, not all initial conditions are consistent with the source $S(t, r)$, since trivial ID violates the jump conditions (13)-(15). Second, jumps $[\Psi]_p$ and $[\phi]_p$ contain

redundant information since, consistently with the definition of ϕ in (7), $[\phi]_p$ is the time derivative of $[\Psi]_p$. However, it is crucial to realize that the jump condition (14) for $[\phi]_p$ does not account for the initial value of $[\Psi]_p$. Therefore, if the evolution scheme fails to implement the condition

$$[\Psi]_p(t_o) = f_p^{-1}F(t_o, r_p), \quad (16)$$

it is no longer equivalent to the model Eq. (1). In [7, 12], this equation is written in first-order form in terms of the variables (Ψ, ϕ, φ) , as in (13)-(15), but the equations for ϕ and φ are now

$$\begin{aligned} \partial_t \phi &= \partial_{r^*} \varphi - V(r)\Psi - J_\varphi \delta(r^* - r_p^*), \\ \partial_t \varphi &= \partial_{r^*} \phi + J_\phi \delta(r^* - r_p^*), \end{aligned} \quad (17)$$

where $J_\varphi = [\varphi]_p$ and $J_\phi = [\phi]_p$. The jump conditions can be recovered by multiplying the equations by a test function, integrating (by parts) between $r^* - \epsilon$ and $r^* + \epsilon$, and taking the limit $\epsilon \rightarrow 0$. However, system (17) does not explicitly enforce the jump condition on $[\Psi]_p$. More precisely, the jump condition on $[\Psi]_p$ is expected to be implemented by enforcing $[\phi]_p$, which means that $[\Psi]_p$ is enforced up to an initial condition: Only the value of $\partial_t [\Psi]_p$ is enforced. Since Ψ is coupled to the rest of the system through the potential term, $V(r)\Psi$, this has consequences on the whole system. In conclusion, the evolution system (17) is not equivalent to Eq. (1).

Another observation is that trivial ID is confusing: From the source perspective we must impose the condition (16), but from the point of view of the ID it seems reasonable to choose $[\Psi]_p(t_o) = 0$. This confusion is just a consequence of the inconsistency between the singular source term and trivial ID that we pointed out above.

The main observation of this work is that Jost junk solutions appear as a consequence of implementing a *finite* jump condition, $[\Psi]_p$, by enforcing an infinitesimal condition in time (the jump differential equation $\partial_t [\Psi]_p = [\phi]_p$), *without* simultaneously imposing the initial value of $[\Psi]_p$ that is consistent with the singular source [Eq. (16)]. Therefore, the prescription to eliminate Jost junk solutions is simple: To enforce the initial value $[\Psi]_p(t_o)$ along the evolution so that the sources are correctly implemented.

In light of this discussion the results and conclusions in [7] are correct. In particular, it is concluded there that Jost junk solutions are not numerical artifacts but rather they are related to the implementation of the singular source term. However, the discussion about the specific reason underlying the appearance of a Jost solution is not conclusive, as the failure to enforce the initial condition (16) is not identified as the underlying cause. Nevertheless, a way of avoiding the problem is proposed in [7], consisting of a redefinition of φ [16]

$$\tilde{\varphi} = \varphi + [\Psi]_p \delta(r^* - r_p^*). \quad (18)$$

Then, the equations for $(\phi, \tilde{\varphi})$ become

$$\begin{aligned} \partial_t \phi &= \partial_{r^*} \tilde{\varphi} - V(r)\Psi - J_\varphi \delta(r^* - r_p^*) - J_\Psi \delta'(r^* - r_p^*), \\ \partial_t \tilde{\varphi} &= \partial_{r^*} \phi, \end{aligned} \quad (19)$$

with $J_\Psi = [\Psi]_p$. No Jost solutions are found in the implementation of these equations with trivial ID for $(\Psi, \tilde{\varphi}, \phi)$, and it is concluded in [7] that such kind of contamination is not *generic*. The reason is clear from the conclusions we have reached: system (19) implements the finite condition (13) on $[\Psi]_p$, including (16), whereas system (17) does not. Actually, system (19) is equivalent to the second-order model Eq. (1), and this statement is independent of any discussion about ID [17]. An alternative way to understand the contrast between systems (17) and (19) is to note that trivial ID for $\tilde{\varphi}$ and φ have (cf. [17]) completely different content. Whereas the ambiguity stemming from the inconsistency between singular sources and trivial ID was resolved in the case of (17) by keeping trivial ID and ignoring the δ' term (by ignoring the value of $[\Psi]_p(t_o)$), in system (19) one instead prioritizes the correct implementation of the singular source from $t = t_o$ at the (minimal) price of modifying the trivial ID on φ through (18) and $\tilde{\varphi}|_{t=t_o} = 0$, which is equivalent to (16). In relation to this we make a statement that we discuss later: Preserving the singular source at late times [and this, in certain implementations of (1) like (17) and (19), depends critically on the correctness of the source at $t = t_o$] systematically removes the Jost junk solution, independently of the ID.

The problem discussed here is generic in the sense that it affects any linear system with distributional Dirac-delta sources in which (i) the sources are translated into jumps of the fields, and (ii) these jumps are implemented infinitesimally in time through an evolution equation. Consistency with the original system of equations demands the explicit enforcement of the initial value of the jump, otherwise a Jost junk solution will be present.

To illustrate this point, we consider the computation of the self force in the simplified scenario corresponding to a charged scalar particle orbiting a nonrotating MBH. The spherical harmonic modes of the retarded scalar field, $\Phi_{\ell m}$, satisfy the model Eq. (1) with the following identifications: $\Psi = r\Phi_{\ell m}$, $F = 0$, and $G = f^{-1}S_{\ell m}$ [see, e.g. [10], for the expressions of $V(r)$ and $S_{\ell m}(t, r)$]. With this, we can use the first-order reduction of Eqs. (8) or the one presented in [11] based on characteristic variables, (Ψ, u, v) , with $(u, v) = (\phi - \varphi, \phi + \varphi)$. Applying the splitting of the PwP approach [Eqs. (9)-(11)] we obtain homogeneous equations and jump conditions:

$$\begin{aligned} \partial_t \Psi^\pm &= (u^\pm + v^\pm)/2, \quad [\Psi]_p(t) = 0, \\ \partial_t u^\pm &= -\partial_{r^*} u^\pm - V(r)\Psi^\pm, \quad [u]_p(t) = -G(t, r_p), \\ \partial_t v^\pm &= \partial_{r^*} v^\pm - V(r)\Psi^\pm, \quad [v]_p(t) = G(t, r_p). \end{aligned} \quad (20)$$

The crucial point here is the implementation of jumps $[u]_p(t)$ and $[v]_p(t)$. In [11] two different strategies have been adopted: (I) Finite jumps $[u]_p(t)$ and $[v]_p(t)$ are

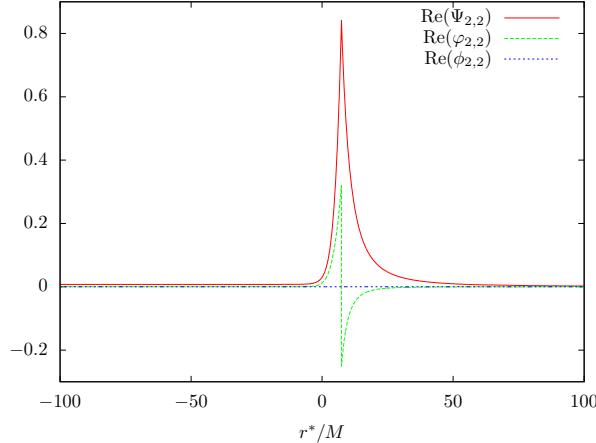


FIG. 1: *Jost solution* built by subtracting solutions to system (20) in approaches (I) and (II) for jump implementation (cf. text), and using trivial ID in *both* cases.

directly enforced in the evolution and trivial ID is used; (II) The time derivatives of the jumps, $d[u]_p/dt$ and $d[v]_p/dt$, are imposed as extra evolution equations using the method of lines. Here, trivial ID cannot be employed and one must “*impose initially the values of the jumps [...] , since during the evolution the only input about them is the information on their derivatives*” (cf. discussion in Sec. V of [11]). Instead, appropriately modified ID is used [note the similarity to the discussion of system (19)].

Therefore, a natural question is whether or not a Jost junk solution appears when taking the difference between the solutions obtained by implementing approaches (I) and (II) but using trivial ID in *both* cases. Figure 1 shows the results obtained from the numerical implementation with the techniques discussed in Ref. [11], which confirm our conclusion that Jost junk solutions correspond to incorrect implementations of the distributional sources, rather than to (trivial) ID inconsistent with the sources.

Hitherto, we have focused mainly on consistency issues *at initial times*. In order to get a deeper insight, we paraphrase the main discussion above in terms of solutions *at late times*. The only relevant element in the analysis at late times is the correct implementation of the sources, the choice of ID playing no role. In particular, any intermediate-time data can be taken as valid ID. From this perspective, the crucial impact of the failure in passing the initial value of the field jumps [18] is the incorrect implementation of the sources, which differ from the correct ones at late times. Then, one is solving a different problem.

The reason why a Jost solution appears when evaluating Ψ_{Jost}^N [Eq. (4)] in [7] is because the smooth time switch-on function, $\alpha(t, \tau)$, guarantees the correct implementation of the late time sources in the solution Ψ_{Smooth} when adopting a strategy in which the source is implemented through an evolution equation for the jumps with zero initial values. To illustrate this, let us con-

sider a field χ with jump condition $[\chi]_p = J_\chi(t)$ and its smoothed version $[\tilde{\chi}]_p = \alpha(t, \tau)J_\chi(t)$. If we implement the jumps using evolution equations, $d[\chi]_p/dt = J'_\chi$ and $d[\tilde{\chi}]_p/dt = (\alpha J_\chi)'$ respectively, both with zero initial values, $[\chi]_p(t_o) = [\tilde{\chi}]_p(t_o) = 0$, we obtain

$$\begin{aligned} [\chi]_p &= J_\chi(t) - J_\chi(t_o), \\ [\tilde{\chi}]_p &= \alpha(t, \tau)J_\chi(t) - \alpha(t_o, \tau)J_\chi(t_o) \\ &= \alpha(t, \tau)J_\chi(t) \simeq J_\chi(t) \quad (\text{for } t \gg \tau). \end{aligned} \quad (21)$$

This shows conclusively that Ψ_{Smooth} solves Eq. (1) at late times with the correct source term, whereas $\Psi_{\text{Impulsive}}$ is contaminated by a Jost solution [cf. point (ii) in the discussion after Eq. (4)].

Time switch on in self-force calculations. Although the smooth switch on is not the critical element in the discussion of the Jost solution [19], it certainly improves the consistency between the source and the ID [which can be critical in certain implementations of the model Eq. (1)]. First, it reduces the initial burst of junk radiation, addressed in detail in [7]. Second, it improves the calculation of the self force in certain implementations. In [7] a similar point was addressed: The calculation of the gauge invariant gravitational-wave fluxes of energy and angular momentum. Regarding the self force itself, we have computed it for the case of a charged scalar particle in circular orbits using the *mode sum* regularization scheme [8]. The retarded field is evolved using the system (20) and trivial ID in the approach (I), i.e. with the direct enforcement of the *finite* jumps $[u]_p$ and $[v]_p$ (this is an example of a correct implementation of the source term while using trivial ID and, as expected, no Jost solution appears). On the other hand, the instantaneous switch on of the source term produces high-frequency numerical noise with a very slow time decay (the decay time scale is much larger than the orbital time scale) and deteriorates significantly the accuracy of the self force. This noise can be eliminated by increasing the resolution of the computations or by using a numerical filter. However, a more efficient and better adapted method is the smooth switch on of the source, which produces dramatic improvements in the computation of the self force. In Fig. 2 we illustrate the calculation of the self force, with and without time switch-on of the sources, for a particle at the last stable circular orbit ($r_p = 6M$) after two orbits, i.e. for $t > 2 \cdot T_{\text{orb}} = 2 \cdot 2\pi(r_p^3/M)^{1/2} \approx 184.69M$ (the self force calculation is physically meaningful at late times, once the initial unphysical burst has been radiated away from the particle). Outgoing boundary conditions discussed in [10, 11] are imposed at the boundaries $[r^*_{\text{inner}} \approx -292M, r^*_{\text{outer}} \approx 307M]$, whose location guarantees their causal disconnection from the particle up to the final evolution time $t_{\text{fin}} = 250M$.

To conclude, we have analyzed the risk of contamination of time-domain self-force calculations by Jost junk solutions as a consequence of using trivial ID inconsistent with the distributional sources, which has produced some

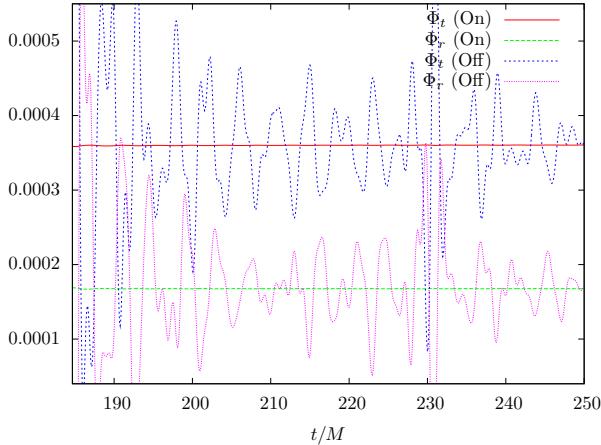


FIG. 2: Radial and time components of the self force as a function of time, with (“On”) and without (“Off”) time switch-on of the sources ($r_p = 6M$). Note that the Off case does not even allow the recovery of the second significant digit in $\Phi_r = 1.677 \cdot 10^{-4}$, $\Phi_t = 3.609 \cdot 10^{-4}$ (see e.g. [10]).

confusion in the community. We have shown that no Jost solution is generated as long as the distributional Dirac-delta sources are faithfully implemented at late times. Indeed, the inconsistency between trivial ID and distributional sources introduces a genuine ambiguity in the evolution system: either one modifies the sources to make them compatible with the trivial ID, or one accepts the use of inconsistent ID and prioritizes the implementation

of the correct sources (at late times). In this specific sense, the conclusion in [7] stating that the presence of Jost solutions is not a numerical artifact, but actually depends on the implementation of the system, is correct. However, it is not a generic feature of these systems and hence can be easily avoided. Here, we have first identified unambiguously the specific origin of Jost solutions in implementations of jump conditions due to Dirac-delta sources, namely, the failure to enforce the correct initial value in the evolution equations for the jumps. Second, our final conclusion is that no contamination of the retarded solution by Jost junk solutions happens as long as (late time) sources are correctly implemented, and this is conceptually independent of the ID and/or the use of a smooth time switch-on. As a by-product of the analysis, we have shown that the use of a smooth time switch-on of the sources [7, 12] prevents the high-frequency noise that spoils self-force calculations in some numerical schemes.

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[13] C. F. Sopuerta and P. Laguna, Phys. Rev. **D73**, 044028 (2006), gr-qc/0512028.
[14] $[\chi]_p$ denotes a jump of a function χ at $r = r_p$, i.e. $[\chi]_p \equiv \lim_{\epsilon \rightarrow 0} (\chi(r_p + \epsilon) - \chi(r_p - \epsilon))$.
[15] Note the opposite sign convention in the definition of ϕ here as compared to Π in [7, 12].
[16] Note that this is precisely the redefinition needed for consistency with (11), in particular for trivial ID for φ^\pm .
[17] Note a secondary source of ambiguity associated with the incorrect implementation of the finite jump $[\Psi]_p$ that renders expression for $[\varphi]_p(t)$ in Eq. (13) of [12] (with $\dot{r}_p = \ddot{r}_p = 0$) nonequivalent to our expression (15), therefore introducing a potential new source of error.
[18] The initial value of the jump is part of the implementation of the sources and hence, it is prior to and independent from the choice of ID. Only in some particular implementations, like the one in [7], both issues are linked.
[19] The critical element is the correct implementation of the source term at late times, the smooth switch on just providing a particular manner of guaranteeing this.